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ON THE REFLECTION OF WATER WAVES BY A BENDED PLATE IN PRESENCE OF SURFACE TENSION

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Abstract : The problem of reflection of water waves by a rigid bended plate, in water of finite depth is considered in the present analysis. Considering the effect of surface tension at the free surface, a simplified perturbational technique followed by Havelock's expansion [1] of water wave potential is applied to solve the problem, analytically, upto first order. For two special shapes of the bended plate, first order corrections to the velocity potential and refection coefficient are obtained.

***Keywords* – linear theory, inviscid liquid, irrotational flow, vertical cliff, source potential.**

I. INTRODUCTION

The present analysis is concerned with the problem of reflection of water waves involving a rigid bended plate in water of finite depth assuming surface tension effect at the free surface. The problem of incoming water waves incident on a vertical cliff and few of its generalization have been considered by some investigators for a long time [2-6]. However, existing literature on problems including surface tension effect at the free surface are, in general complicated. Since then attempts have been made to study this type of water of problems by employing different mathematical techniques [7-11].

However, very few attempts have been reported so far to study the problem involving a bended plate. The first problem in this area has been tackled by Shaw [12] where he used a perturbation technique that involves the solution of a singular integral equation to find the first order corrections to the reflection and transmission coefficients in connection with a surface piercing nearly vertical barrier in deep water. Since then, some attempts have been made to study this class of water wave problem and few of its' generalizations by employing different mathematical methods [12-15].

In the present paper, the problem under consideration is attacked for solution by a simplified perturbation analysis followed by an appropriate Havelock's expansion of water wave potential. Corrections up to first order, for the reflection co-efficient as well as the velocity potential are obtained in terms of an integral involving the shape function for finite depth of water in presence of surface tension. Finally these are calculated explicitly by assuming two special shapes of the bended plate.

II. FORMULATION OF THE PROBLEM

Cartesian co-ordinates are selected in which y-axis measured vertically downwards and assume that the water is bounded on the left side by the bended plate $x = \varepsilon f(y), 0 < y < a \ (0 < \varepsilon \ll 1)$ where $f(y)$ is a bounded and continuous function with $f(0) = 0$ and below by a plane bottom $y = a$ so that $y = 0, x > 0$ is the unimpeded free surface.

As usual assumption of the linearized water wave theory is adopted and the motion is irrotational, which ensure the existence of a velocity potential

$$\Phi(x,y,t) = Re[\phi(x,y)\exp(-i\omega t)]$$

where ω is the circular frequency of the incident waves. Thus the problem under consideration can be investigated by way of determining the potential function $\phi(x,y)$, satisfying the following boundary value problem (BVP):

(i) The equation of continuity generates Laplace's equation:

$$\nabla^2\phi = 0 \text{ in the fluid region.} \tag{2.1}$$

(ii) The Linearized structure of kinematic condition at the free surface:

$$K\phi + \phi_y + M\phi_{yyy} = 0 \text{ on } y = 0, x > 0, \tag{2.2}$$

where $K = \omega^2/g$ and $M = \tau/(\rho g)$; τ is the coefficient of surface tension, g is the acceleration due to gravity and ρ is the density of the liquid.

(iii) Rigid body condition:

$$\phi_n = 0 \text{ on } x = \varepsilon f(y), 0 < y < a, \tag{2.3}$$

where n is the unit normal to the surface of the bended plate.

(iv) Bottom condition:

$$\phi_y = 0 \text{ on } y = a. \tag{2.4}$$

Further ϕ is also required to satisfy the following:

$$\phi \sim \frac{\cosh\gamma_0(a-y)}{\cosh\gamma_0 a} exp(-i\gamma_0 x) + R\frac{\cosh\gamma_0(a-y)}{\cosh\gamma_0 a} exp(i\gamma_0 x) \text{ as } x \to \infty \qquad (2.5)$$

where a train of surface waves narrated by $\frac{\cosh\gamma_0(a-y)}{\cosh\gamma_0 a} exp(-i\gamma_0 x)$ is normally incident from positive infinity on the bended plate, R is the reflection coefficient and γ_0 is the unique real positive root of the transcendental equation (cf.[5])

$$x(1+Mx^2)\tanh ax - K = 0.$$

As the parameter ε is very small, thus neglecting $O(\varepsilon^2)$ terms, the boundary condition (2.3) can be approximately expressed on $x = 0$ as (cf. [12],[13])

$$\phi_x(0,y) = \varepsilon\frac{d}{dy}\{f(y)\,\phi_y(0,y)\} \quad \text{for } 0 < y < a. \qquad (2.6)$$

III. Solution By Perturbation Method

The approximate boundary condition (2.6) suggests that we may adopt perturbational expansion in terms of the small parameter ε for the function $\phi(x,y)$ and the unknown constant R respectively as

$$\left.\begin{aligned} \phi(x,y;\varepsilon) &= \phi_0(x,y) + \varepsilon\phi_1(x,y) + O(\varepsilon^2) \\ R(\varepsilon) &= R_0 + \varepsilon R_1 + O(\varepsilon^2). \end{aligned}\right\} \qquad (3.1)$$

Present analysis is confined with the determination of ϕ_0, R_0 and ϕ_1, R_1, as we are attracted in evaluating only the corrections to the velocity potential and reflection coefficient up to first order. Substituting the expansion (3.1) into the equations (2.1), (2.2), (2.4), (2.5) and (2.6) and equating the coefficients of identical powers of ε^0 and ε on both sides, we see that the functions ϕ_0 and ϕ_1 must be the solution of the following two independent BVPs:

BVP-I: The function ϕ_0 satisfies

$$\nabla^2\phi_0 = 0 \text{ in the fluid region,}$$

$$K\phi_0 + \phi_{0_y} + M\phi_{0_{yyy}} = 0 \text{ on } y = 0, x > 0,$$

$$\phi_{0_x} = 0 \text{ on } x = 0, 0 < y < a,$$

$$\phi_{0_y} = 0 \text{ on } y = a,$$

$$\phi \sim \frac{\cosh\gamma_0(a-y)}{\cosh\gamma_0 a} exp(-i\gamma_0 x) + R_0\frac{\cosh\gamma_0(a-y)}{\cosh\gamma_0 a} exp(i\gamma_0 x) \text{ as } x \to \infty.$$

Obviously,

$$\phi_0 = \frac{2\cosh\gamma_0(a-y)}{\cosh\gamma_0 a}\cos\gamma_0 x \qquad (3.2)$$

so that we find $R_0 = 1$.

BVP-II: The function ϕ_1 satisfies

$$\nabla^2\phi_1 = 0 \text{ in the fluid region}$$

$$K\phi_1 + \phi_{1_y} + M\phi_{1_{yyy}} = 0 \text{ on } y = 0, x > 0,$$

$$\phi_{1_x} = \frac{d}{dy}\{f(y)\phi_{0_y}\} \text{ on } x = 0, 0 < y < a, \qquad (3.3)$$

$$\phi_{1_y} = 0 \text{ on } y = a,$$

$$\phi \sim R_1\frac{\cosh\gamma_0(a-y)}{\cosh\gamma_0 a} exp(i\gamma_0 x) \text{ as } x \to \infty.$$

Assume that

$$\frac{d}{dy}\{f(y)\phi_{0_y}\} = g(y) \text{ on } x = 0, 0 < y < a \qquad (3.4)$$

so that, from the boundary condition (3.3), we have

$$\phi_{1_x} = g(y) \text{ on } x = 0, 0 < y < a. \qquad (3.5)$$

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By Havelock's expansion [1], $\phi_1(x,y)$ has the representation

$$\phi(x,y) = R_1 \frac{\cosh \gamma_0(a-y)}{\cosh \gamma_0 a} exp(i\gamma_0 x) + \sum_n B_n cos\, \gamma_n(a-y) exp(-\gamma_n x) \qquad (3.6)$$

where the summation extends over the real positive roots of $K + x(1 - Mx^2)\tan ka = 0$.
Exploiting the boundary condition (3.5) we find

$$g(y) = i\gamma_0 R_1 \frac{\cosh \gamma_0(a-y)}{\cosh \gamma_0 h} - \sum_n \gamma_n B_n \cos \gamma_n(a-y)$$

so that following Rhodes-Robinson[8] we have

$$R_1 = \frac{-4i(1+M\gamma_0^2)\cosh \gamma_o a \int_0^a g(x) \cosh \gamma_0(a-x)\, dx}{2\gamma_0 a(1+M\gamma_0^2) + (1+3M\gamma_0^2)\sinh 2\gamma_o a} \qquad (3.7)$$

and

$$B_n = \frac{-4(1-M\gamma_n^2)\int_0^a g(x)\cos \gamma_n(a-x)\, dx}{2\gamma_n a(1-M\gamma_n^2) + (1-3M\gamma_n^2)\sin 2\gamma_n a}. \qquad (3.8)$$

Thus R_1 and B_n are found when $f(y)$ is known and hence we find the general expression for R_1 and ϕ_1, the first order corrections to the reflection co-efficient R and velocity potential ϕ, in presence of surface tension.

IV. Special Shapes of the Bended Plate

To illustrate the results obtained here, we consider the following two particular shapes of the bended plate:

i) $\boldsymbol{f(y) = y\, exp(-\lambda y), 0 \le y \le a}$

Noting (3.2), we have

$$\phi_0(0,y) = \frac{2\cosh \gamma_0(a-y)}{\cosh \gamma_0 a} \qquad (4.1)$$

Using (4.1) into (3.4) we find

$$g(y) = -\frac{2\gamma_0 \exp(-\lambda y)}{\cosh \gamma_0 a}[(1-\lambda y)\sinh \gamma_0(a-y) - \gamma_0 y \cosh \gamma_0(a-y)]$$

so that from (3.7) and (3.8) we obtain

$$R_1 = \frac{-4i\gamma_0^2(1+M\gamma_0^2)}{\{2\gamma_0 a(1+M\gamma_0^2) + (1+3M\gamma_0^2)\sinh 2\gamma_0 a\}(\lambda^2 - 4\gamma_0^2)^2} \times \left[\frac{1}{\lambda^2}(\lambda^2 - 4\gamma_0^2)^2 + 8\gamma_0^2 \exp(-\lambda a) - (\lambda^2 + 4\gamma_0^2)\cosh 2\gamma_0 a + 4\lambda\gamma_0 \sinh 2\gamma_0 a\right]$$

and

$$\begin{aligned} B_n = &\frac{4\gamma_0(1-M\gamma_n^2)\exp(-\lambda a)}{\cosh \gamma_0 a\{2\gamma_n a(1-M\gamma_n^2) + (1-3M\gamma_n^2)\sin 2\gamma_n a\}} \\ &\times \left[\frac{\lambda+\gamma_0}{\{(\lambda+\gamma_0)^2 + \gamma_n^2\}^2}\{(\lambda+\gamma_0)a((\lambda+\gamma_0)^2 + \gamma_n^2) - 2\gamma_n^2\}\right. \\ &- \frac{\lambda-\gamma_0}{\{(\lambda-\gamma_0)^2 + \gamma_n^2\}^2}\{(\lambda-\gamma_0)a((\lambda-\gamma_0)^2 + \gamma_n^2) - 2\gamma_n^2\} \\ &+ \frac{\gamma_n \exp\{(\lambda+\gamma_0)a\}}{\{(\lambda+\gamma_0)^2 + \gamma_n^2\}^2}\{2(\lambda+\gamma_0)\gamma_n \cos \gamma_n a - ((\lambda+\gamma_0)^2 - \gamma_n^2)\sin \gamma_n a\} \\ &\left. - \frac{\gamma_n \exp\{(\lambda-\gamma_0)a\}}{\{(\lambda-\gamma_0)^2 + \gamma_n^2\}^2}\{2(\lambda-\gamma_0)\gamma_n \cos \gamma_n a - ((\lambda-\gamma_0)^2 - \gamma_n^2)\sin \gamma_n a\}\right] \end{aligned}$$

ii) $\boldsymbol{f(y) = c\sinh(\lambda y), 0 \le y \le a}$

Using (4.1) into (3.4) we find

$$g(y) = \frac{-c\gamma_0}{\cosh \gamma_0 a}[(\lambda-\gamma_0)\sinh\{(\lambda-\gamma_0)y + \gamma_0 a\} - (\lambda+\gamma_0)\sinh\{(\lambda+\gamma_0)y - \gamma_0 a\}]$$

so that from (3.7) and (3.8) we obtain

$$R_1 = \frac{-4ic\gamma_0(1+M\gamma_0^2)}{2\gamma_0 a(1+M\gamma_0^2)+(1+3M\gamma_0^2)\sinh 2\gamma_0 a}\left[1+\frac{\lambda\gamma_0\cosh 2\gamma_0 a-(\lambda^2+\lambda\gamma_0-4\gamma_0^2)\cosh\lambda a}{\lambda^2-4\gamma_0^2}\right]$$

and

$$B_n = \frac{8c\lambda\gamma_0\gamma_n(1-M\gamma_n^2)}{\cosh\gamma_0 a}\{2\gamma_n a(1-M\gamma_n^2)+(1-3M\gamma_n^2)\sin 2\gamma_n a\}^{-1}\{(\lambda-\gamma_0)^2+\gamma_n^2\}^{-1}\{(\lambda+\gamma_0)^2+\gamma_n^2\}^{-1}$$
$$\times\,[2\gamma_0(\cosh\lambda a-\cosh\gamma_0 a\cos\gamma_n a)+(\lambda^2-4\gamma_0^2+\gamma_n^2)\sinh\gamma_0 a\sin\gamma_n a]$$

V. DISCUSSION

A simple perturbation technique along with the application of Havelock's expansion is employed here to find the first order corrections to the reflection coefficient and velocity potential for the reflection of surface water waves incident on a bended plate in finite depth of water, in presence of surface tension at the free surface. Analytical expressions for these corrections are also obtained by assuming two particular shape of the bended plate viz.$(i) f(y) = y\exp(-\lambda y), 0 \leq y \leq a$ $(ii) f(y) = c\sinh\lambda y, 0 \leq y \leq a$.

It should be noted here that in absence of surface tension effect, the approximate solution of the corresponding problem can be found, by the substitution of $\tau = 0$. The problem discussed in the present paper seems to have some applications in coastal design criteria and to derive the solution of the problem considered here, total reflection of waves by the bended plate is assumed since there is no mechanism to absorb the incoming energy in the inviscid fluid system. Thus the reflection of waves is a physically possible phenomenon in any nondissipating system.

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