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ON THE REFLECTION OF WATER WAVES BY A BENDED PLATE IN PRESENCE OF SURFACE TENSION

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Abstract : The problem of reflection of water waves by a rigid bended plate, in water of finite depth is considered in the present analysis. Considering the effect of surface tension at the free surface, a simplified perturbational technique followed by Havelock's expansion [1] of water wave potential is applied to solve the problem, analytically, upto first order. For two special shapes of the bended plate, first order corrections to the velocity potential and refection coefficient are obtained.

***Keywords* – linear theory, inviscid liquid, irrotational flow, vertical cliff, source potential.**

I. INTRODUCTION

The present analysis is concerned with the problem of reflection of water waves involving a rigid bended plate in water of finite depth assuming surface tension effect at the free surface. The problem of incoming water waves incident on a vertical cliff and few of its generalization have been considered by some investigators for a long time [2-6]. However, existing literature on problems including surface tension effect at the free surface are, in general complicated. Since then attempts have been made to study this type of water of problems by employing different mathematical techniques [7-11].

However, very few attempts have been reported so far to study the problem involving a bended plate. The first problem in this area has been tackled by Shaw [12] where he used a perturbation technique that involves the solution of a singular integral equation to find the first order corrections to the reflection and transmission coefficients in connection with a surface piercing nearly vertical barrier in deep water. Since then, some attempts have been made to study this class of water wave problem and few of its' generalizations by employing different mathematical methods [12-15].

In the present paper, the problem under consideration is attacked for solution by a simplified perturbation analysis followed by an appropriate Havelock's expansion of water wave potential. Corrections up to first order, for the reflection co-efficient as well as the velocity potential are obtained in terms of an integral involving the shape function for finite depth of water in presence of surface tension. Finally these are calculated explicitly by assuming two special shapes of the bended plate.

II. FORMULATION OF THE PROBLEM

Cartesian co-ordinates are selected in which y-axis measured vertically downwards and assume that the water is bounded on the left side by the bended plate $x = \varepsilon f(y), 0 < y < a$ $(0 < \varepsilon \ll 1)$ where $f(y)$ is a bounded and continuous function with $f(0) = 0$ and below by a plane bottom $y = a$ so that $y = 0, x > 0$ is the unimpeded free surface.

As usual assumption of the linearized water wave theory is adopted and the motion is irrotational, which ensure the existence of a velocity potential

$$\Phi(x, y, t) = Re[\phi(x, y)\exp(-i\omega t)]$$

where ω is the circular frequency of the incident waves. Thus the problem under consideration can be investigated by way of determining the potential function $\phi(x, y)$, satisfying the following boundary value problem (BVP):

(i) The equation of continuity generates Laplace's equation:

$$\nabla^2\phi = 0 \text{ in the fluid region.} \quad (2.1)$$

(ii) The Linearized structure of kinematic condition at the free surface:

$$K\phi + \phi_y + M\phi_{yyy} = 0 \text{ on } y = 0, x > 0, \quad (2.2)$$

where $K = \omega^2/g$ and $M = \tau/(\rho g)$; τ is the coefficient of surface tension, g is the acceleration due to gravity and ρ is the density of the liquid.

(iii) Rigid body condition:

$$\phi_n = 0 \text{ on } x = \varepsilon f(y), 0 < y < a, \quad (2.3)$$

where n is the unit normal to the surface of the bended plate.

(iv) Bottom condition:

$$\phi_y = 0 \text{ on } y = a. \quad (2.4)$$